Low-depth amplitude estimation via statistical eigengap estimation

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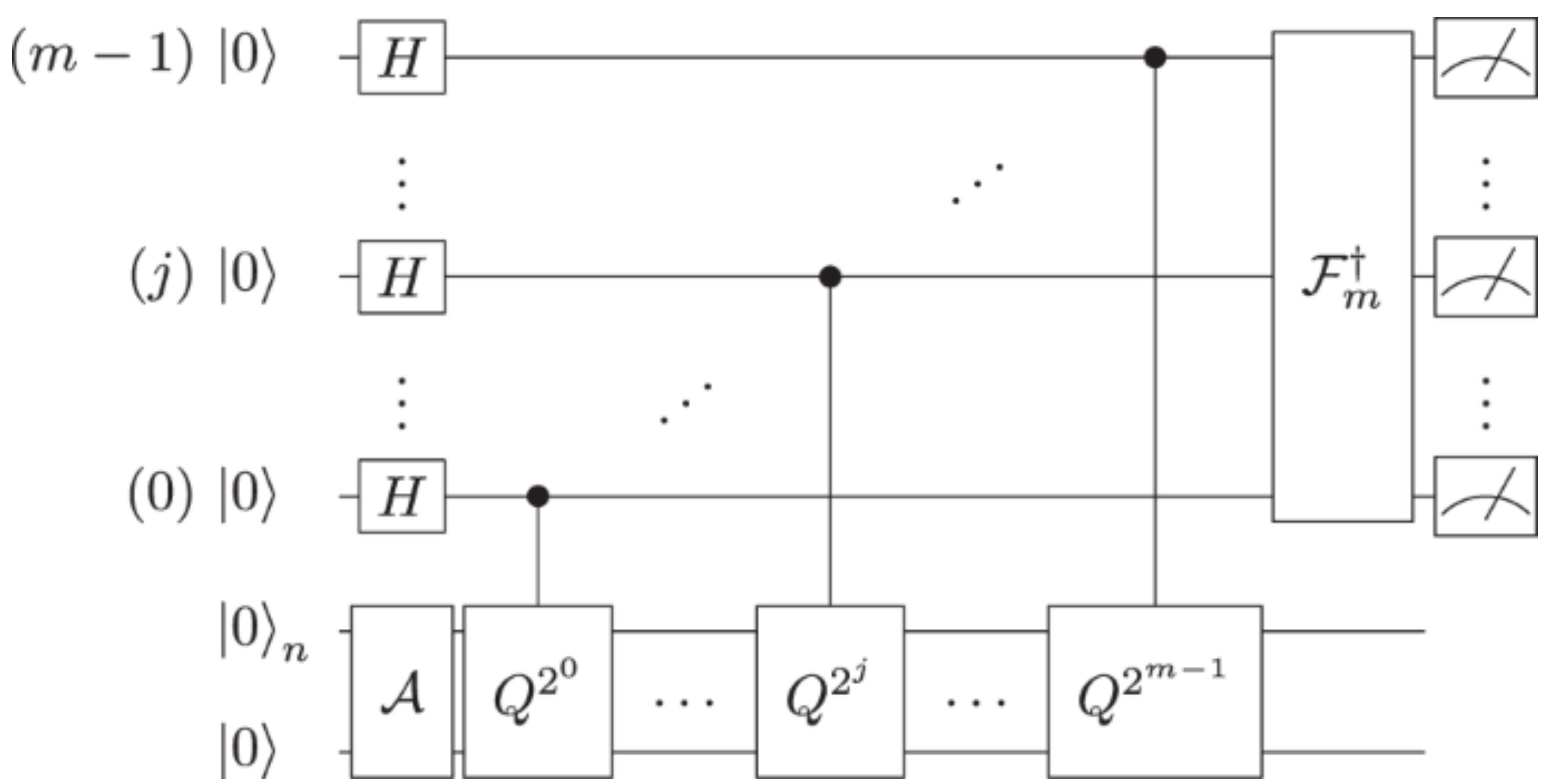
arXiv: 2603.05475

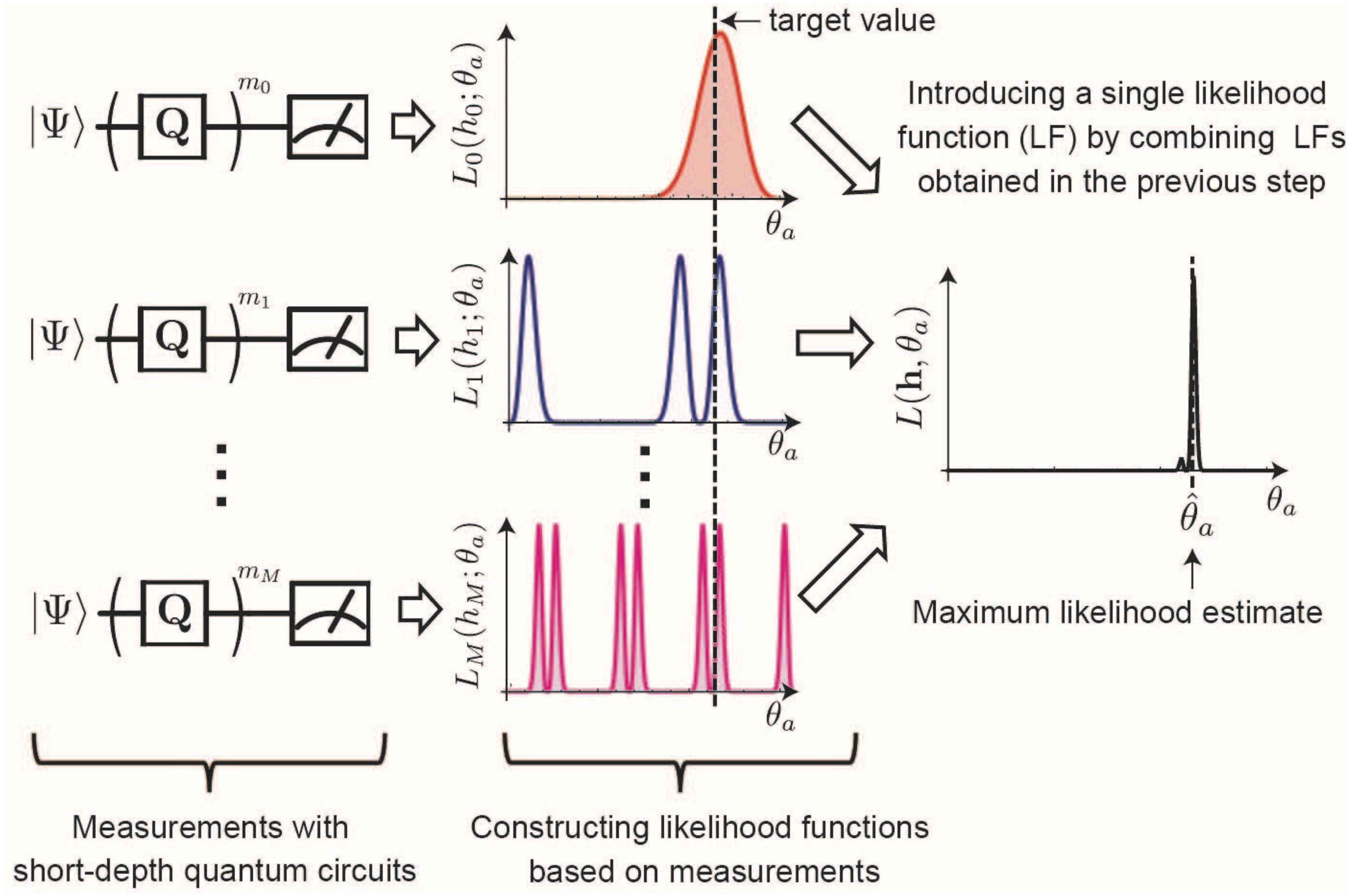
University of Oxford

Quantum Motion

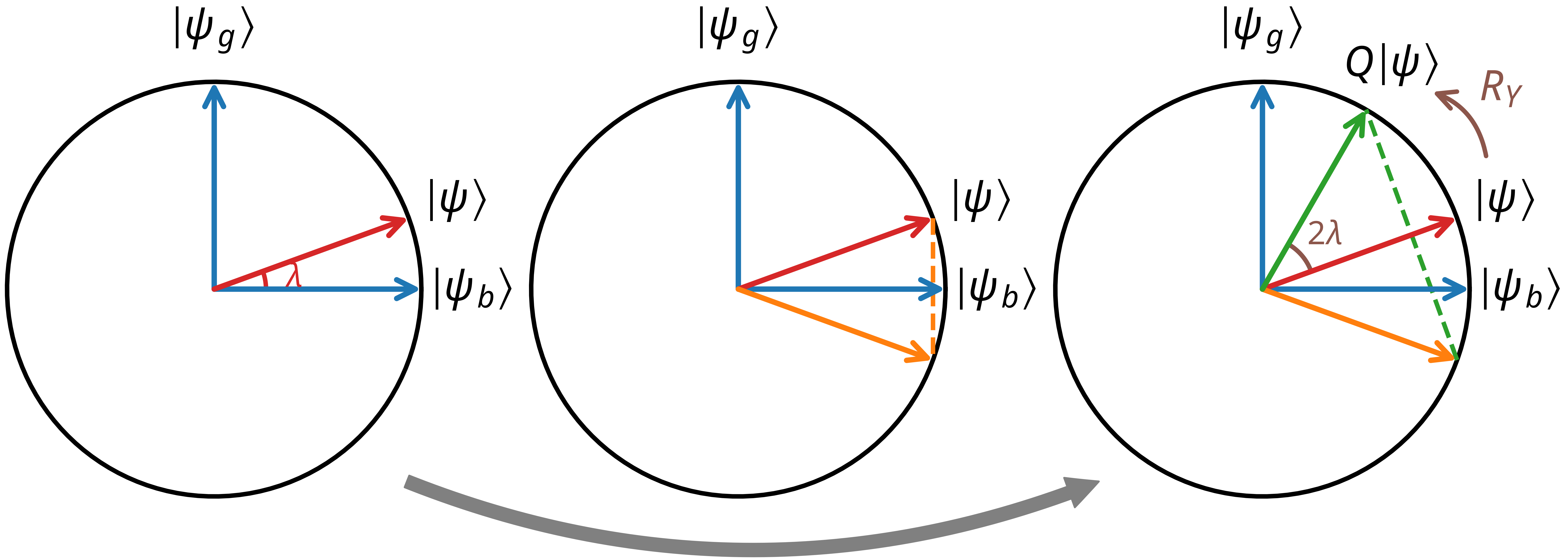
QAE does not need QPE

- Modern QAE protocols only require amplitude amplification runs without QPE.



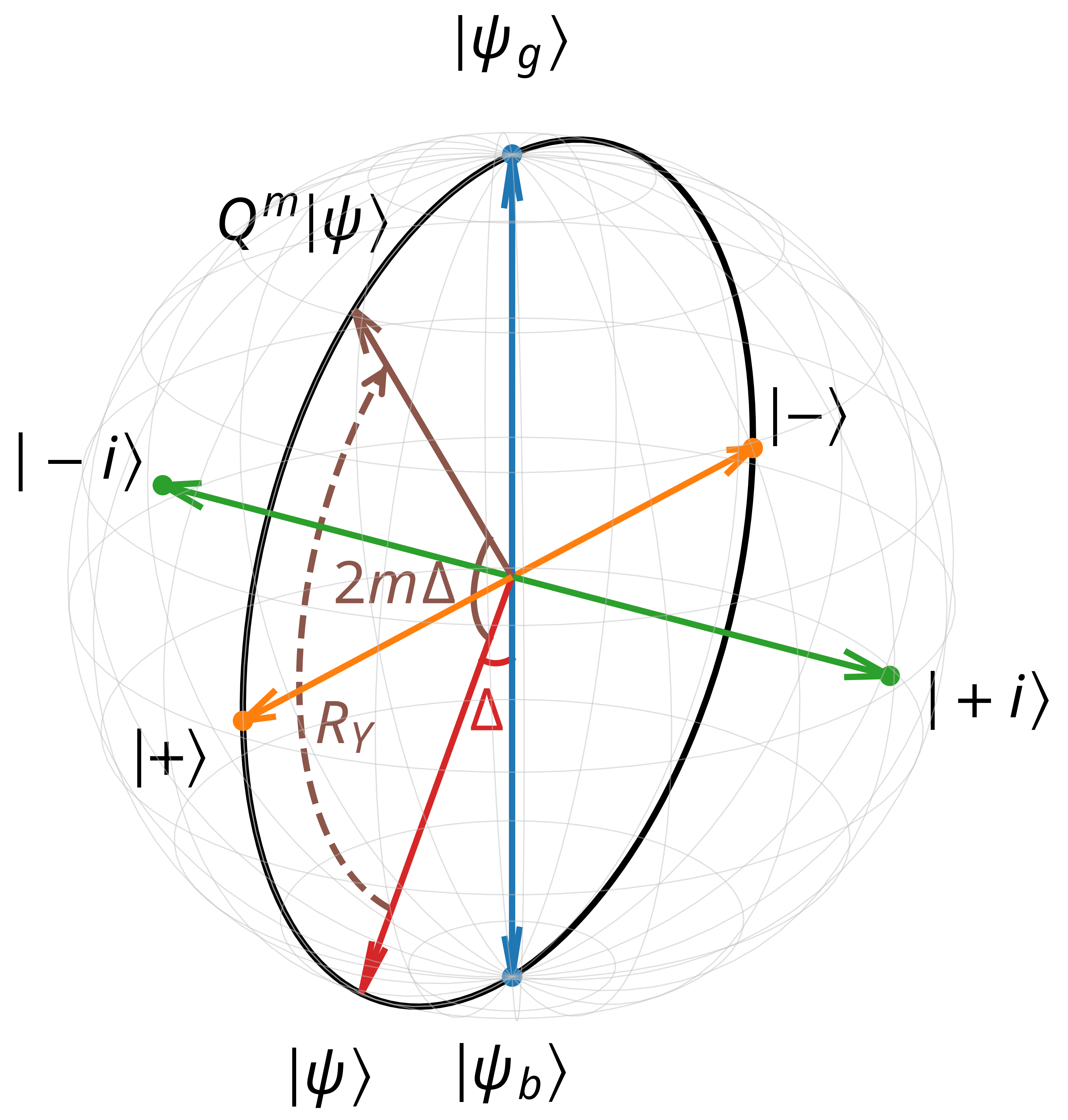


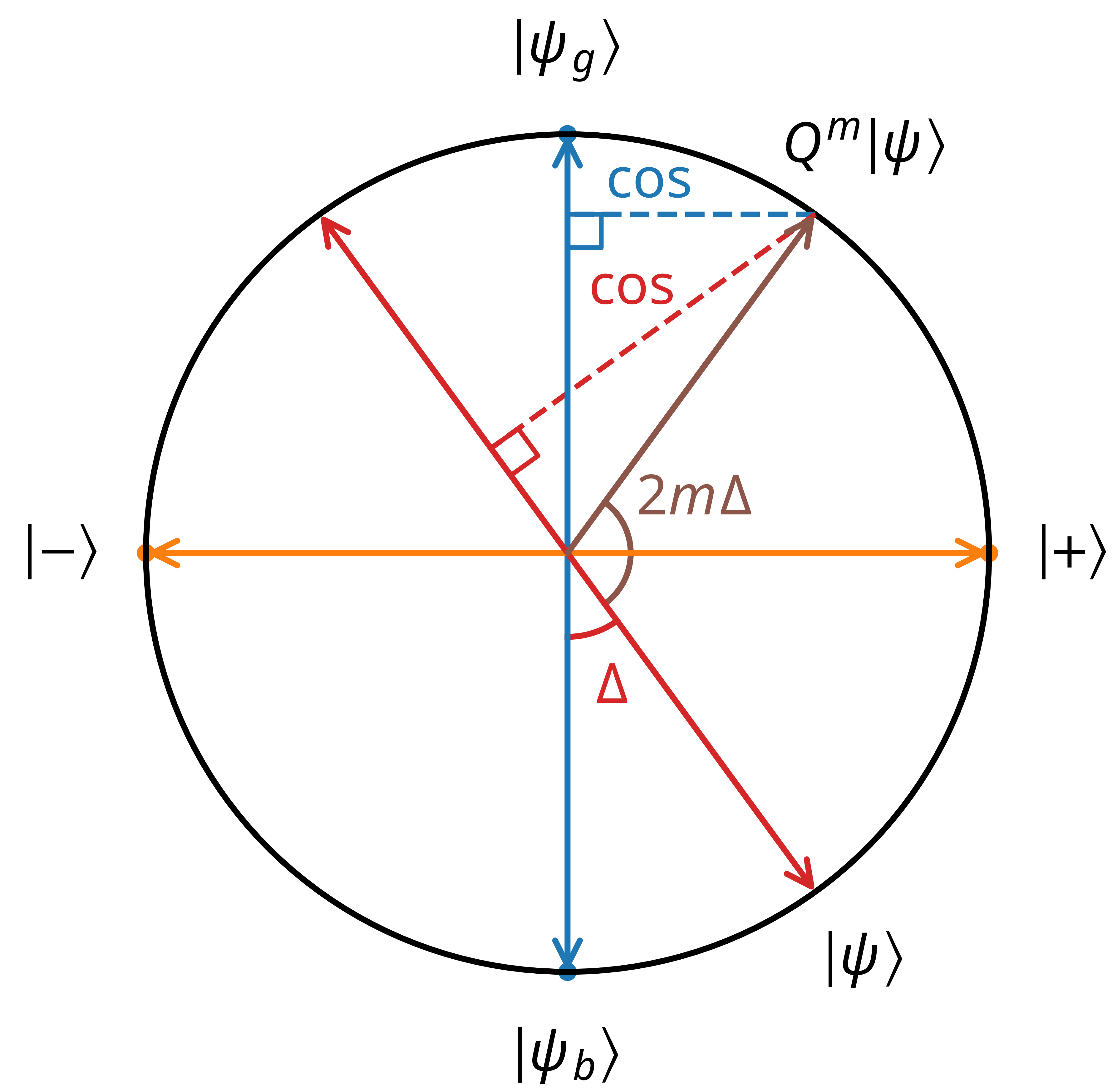
Amplitude amplification as discrete-time evolution

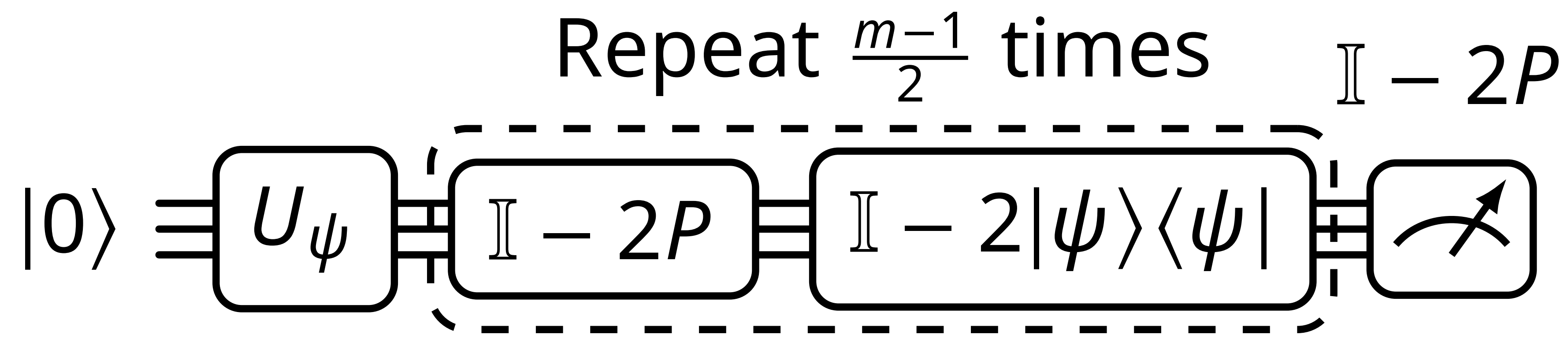


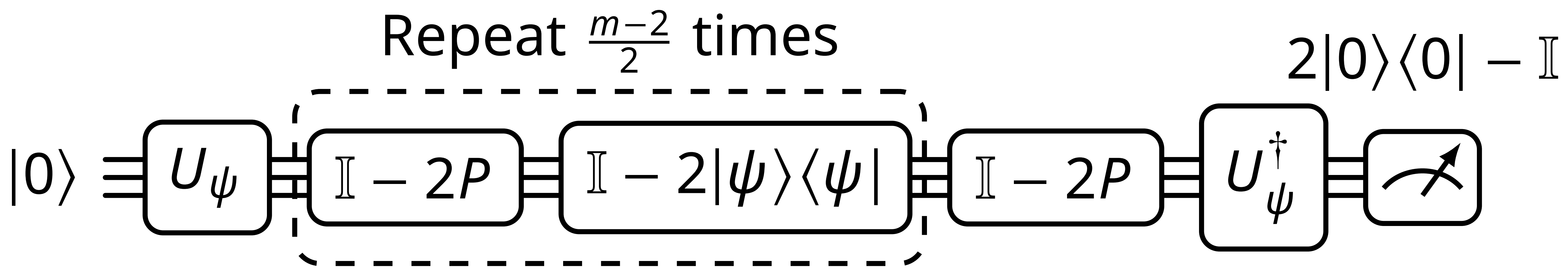
$Q = e^{-iH_{\mathrm{eff}}t}$ (Pauli Y rotation in subspace)

Amplitude estimation as eigengap estimation

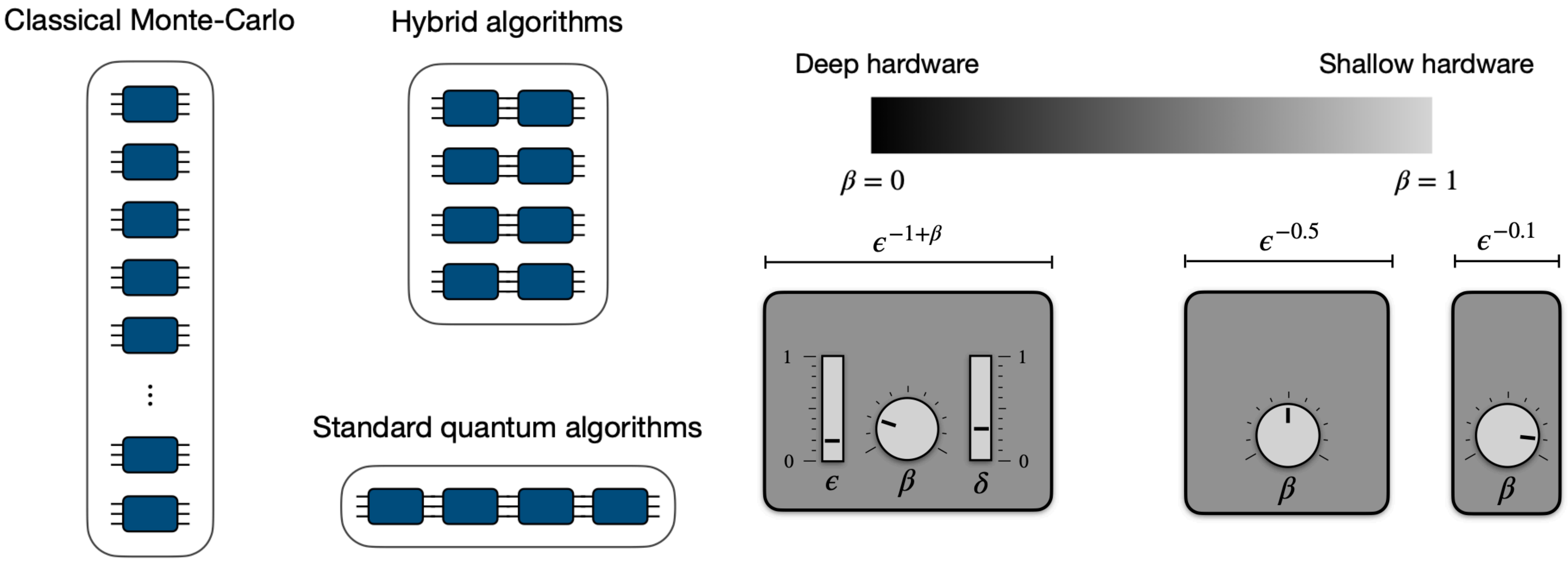






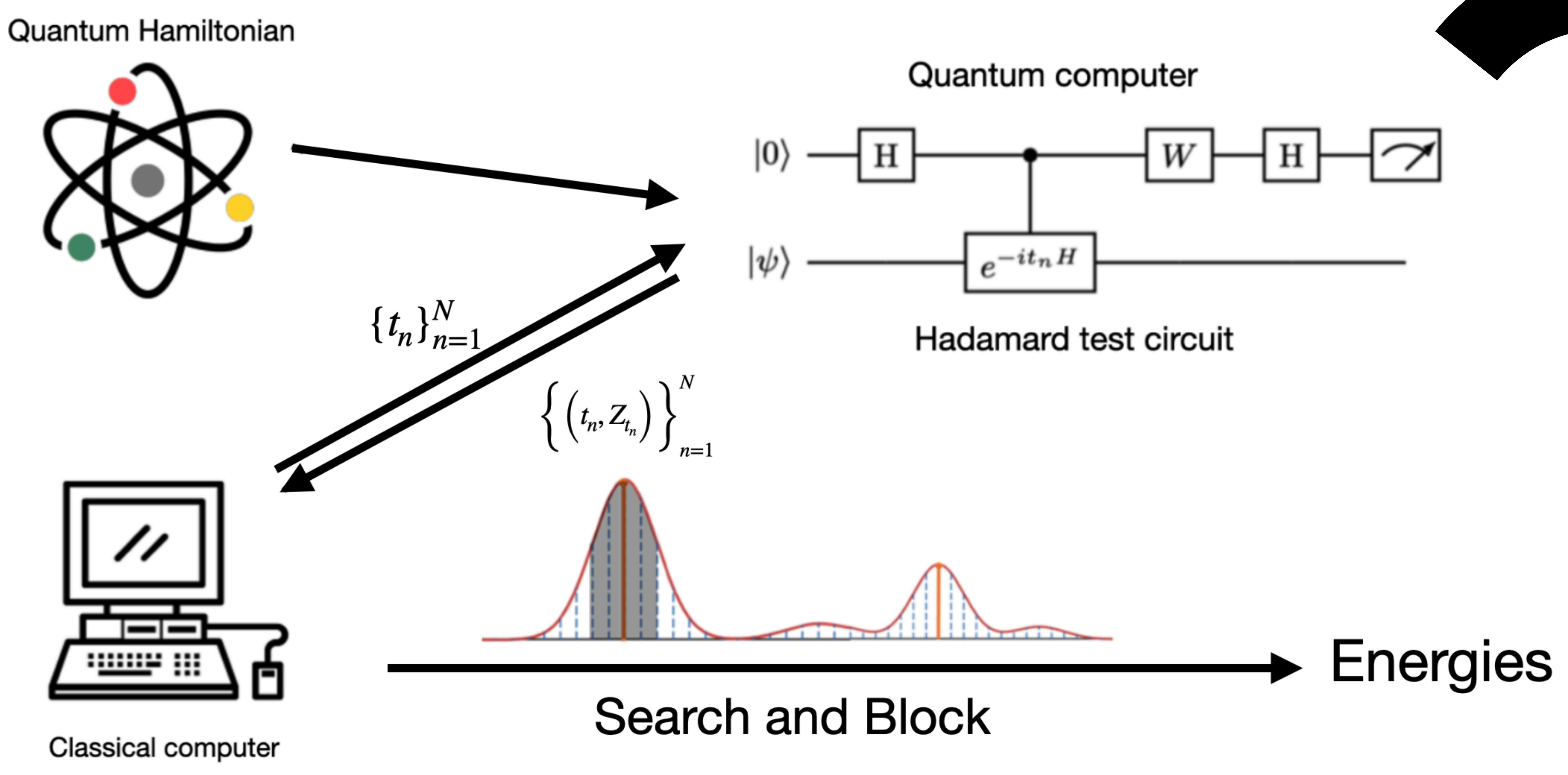


Low-depth amplitude estimation

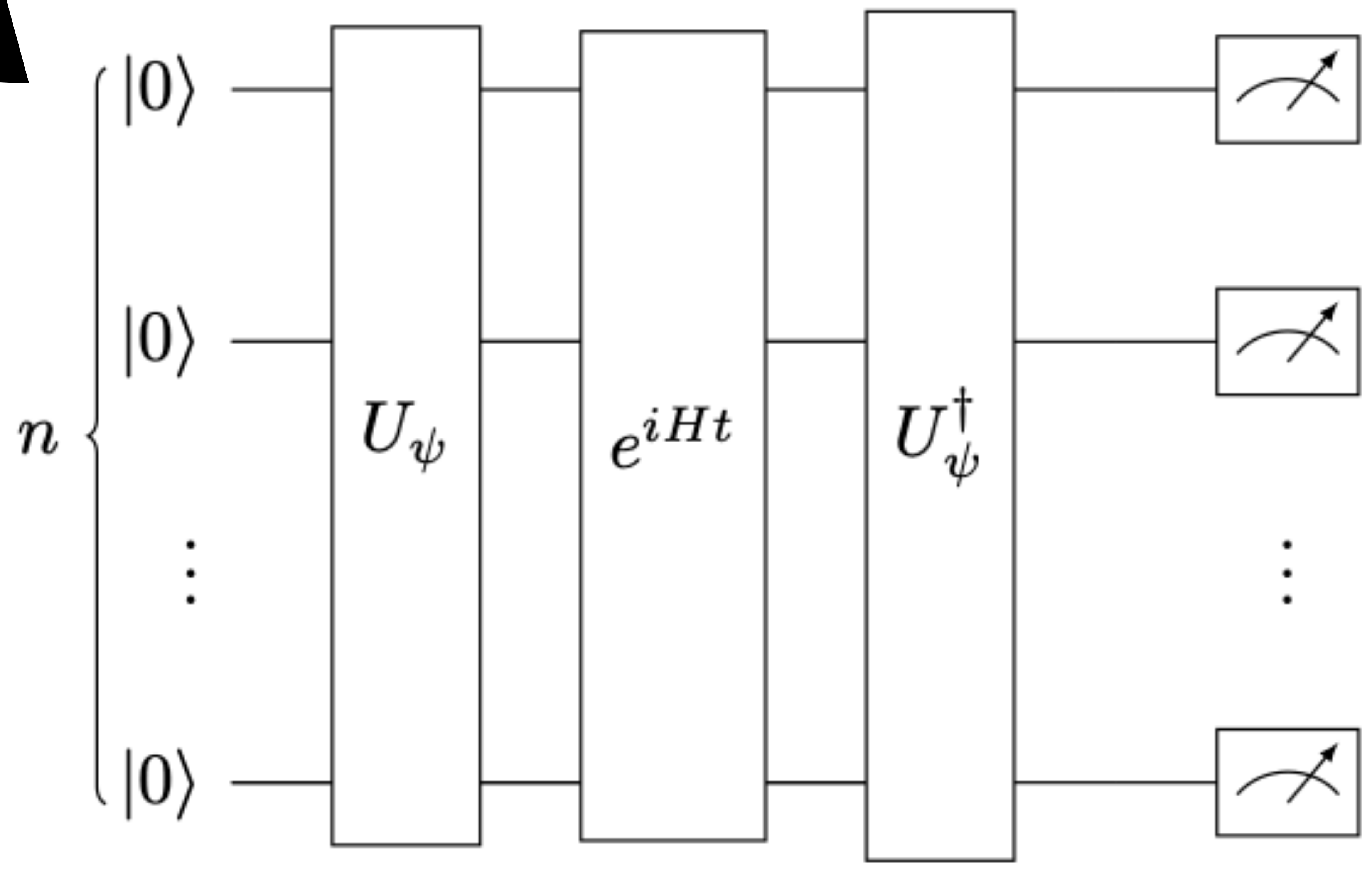


From statistical phase estimation to QAE

Replace Hadamard Test



Statistical Phase Estimation



Eigengap Estimation

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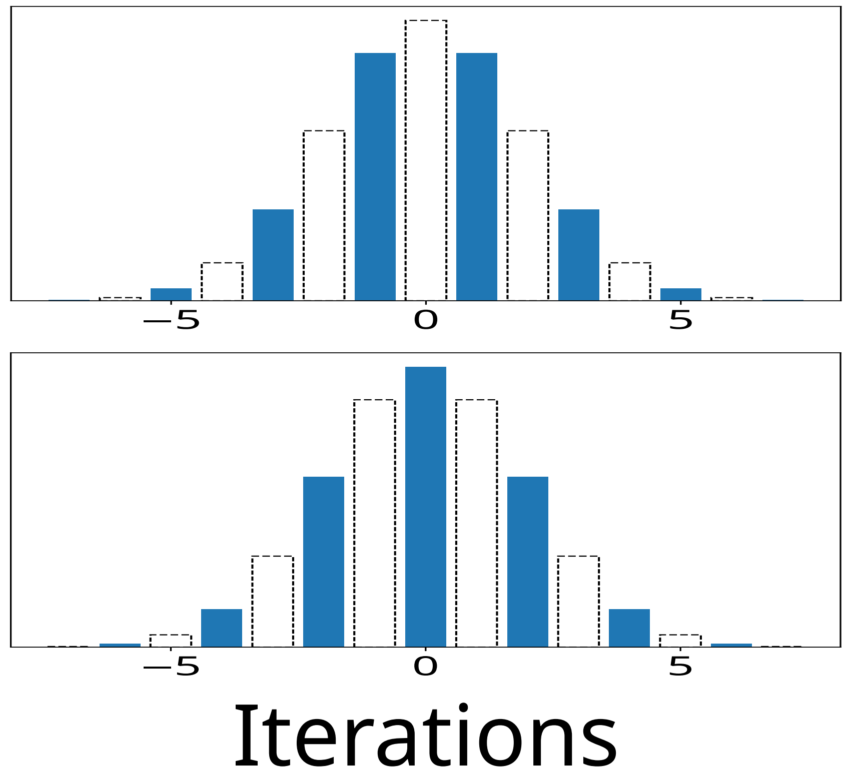
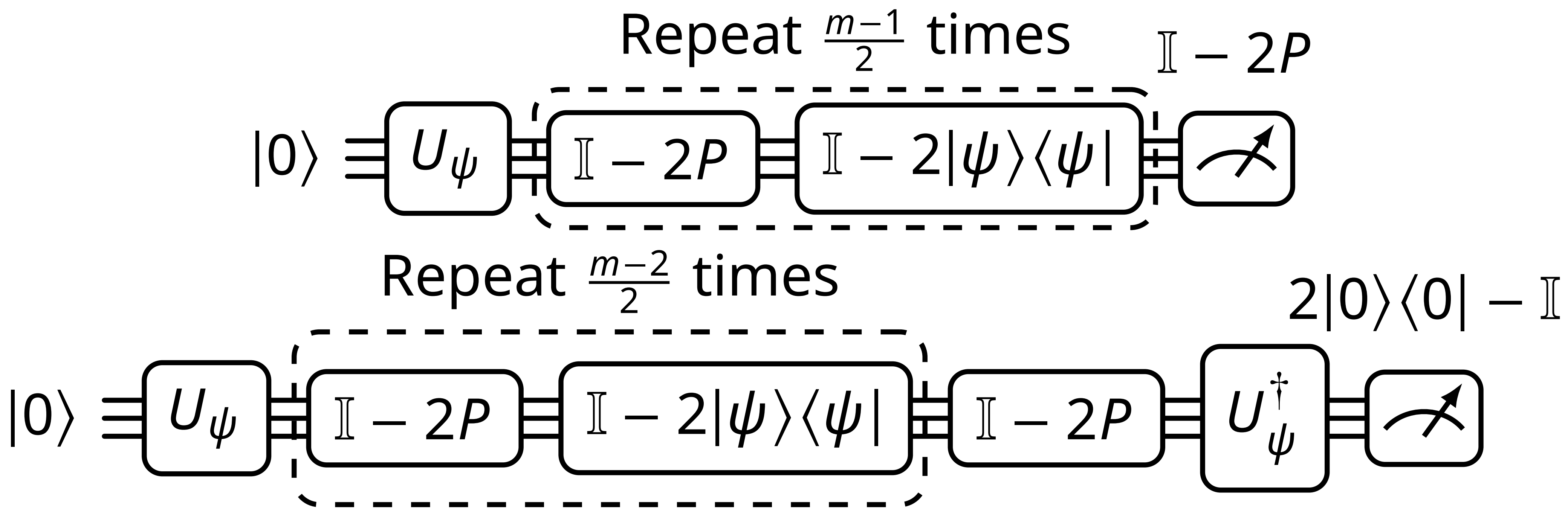
Amplitude Estimation

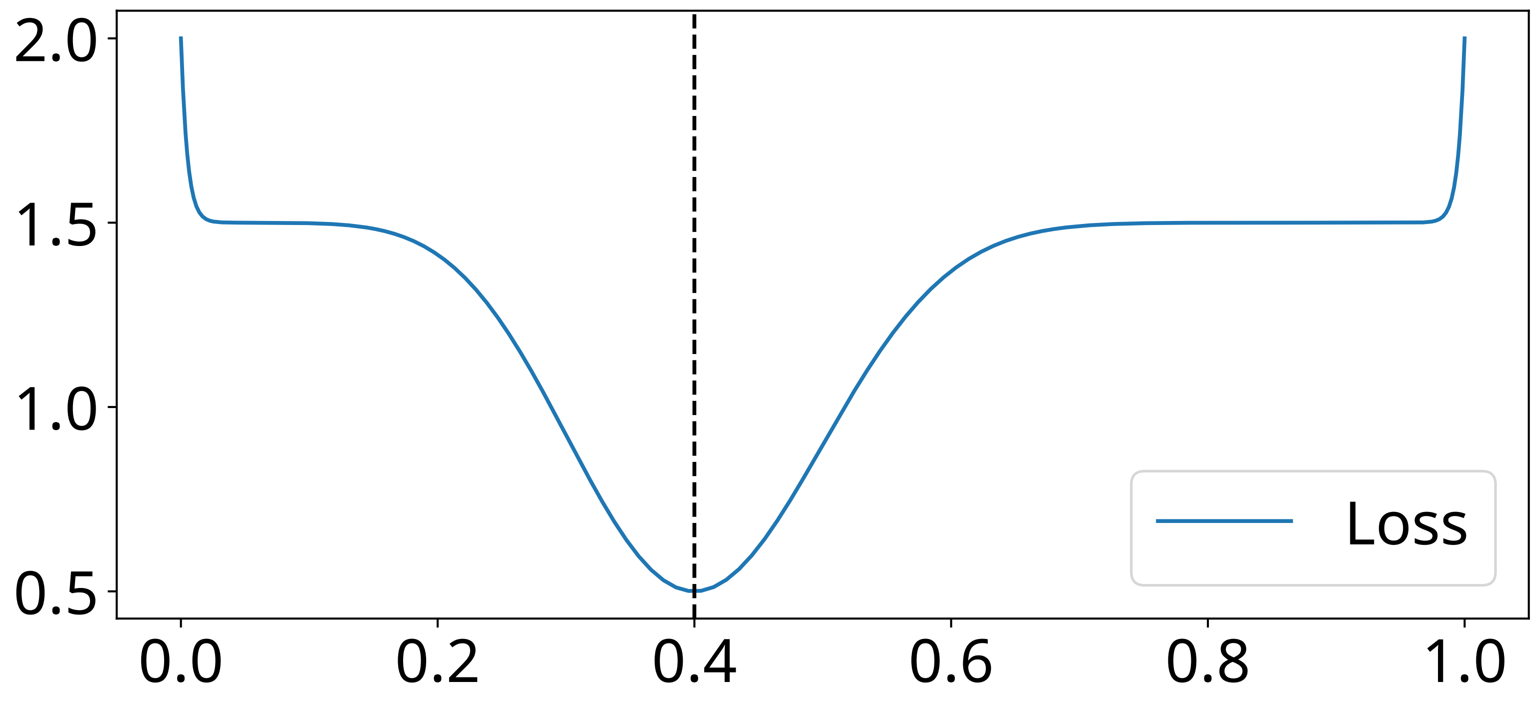
Gaussian Least Squares Amplitude Estimation



$\{m\}_{n\in[N]}$

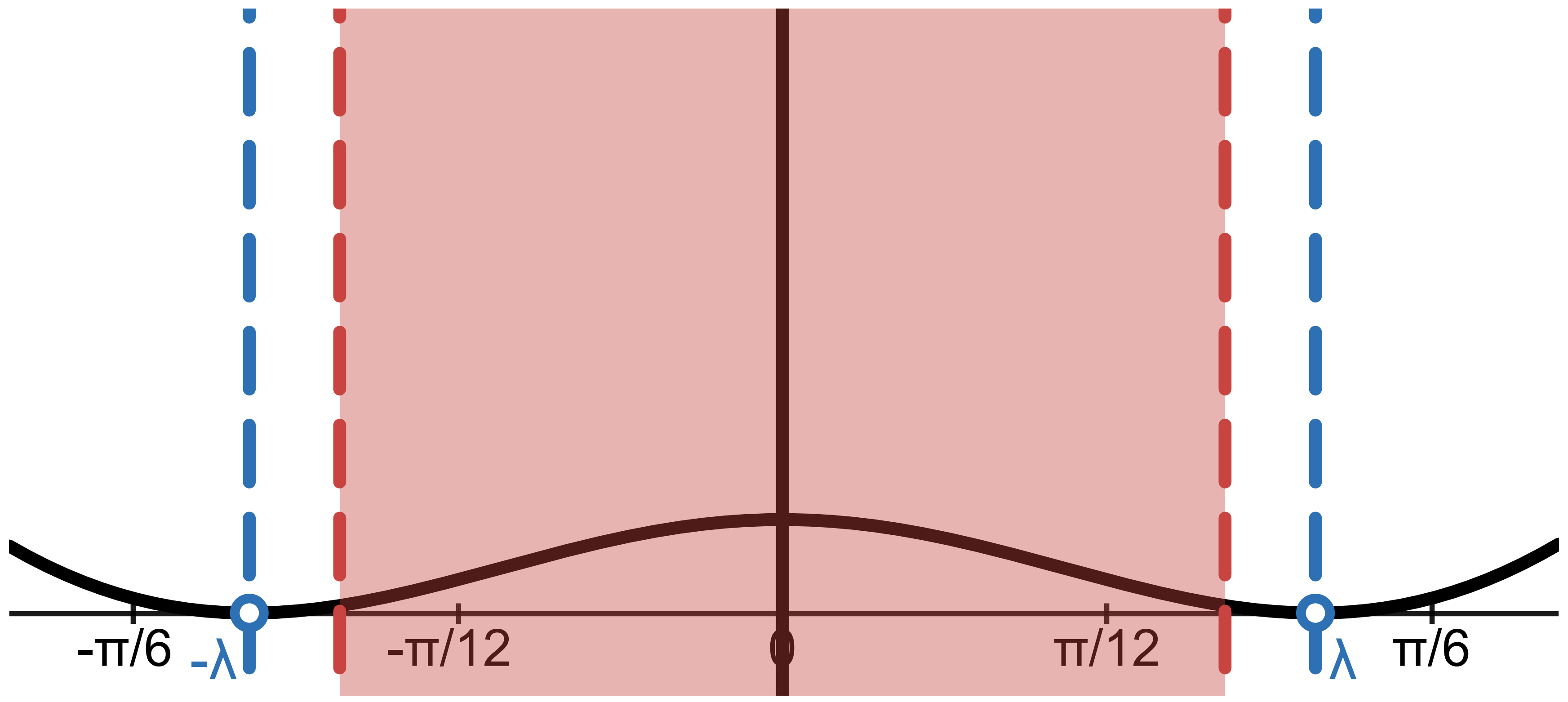


$\{(m, Z_m)\}_{n\in[N]}$

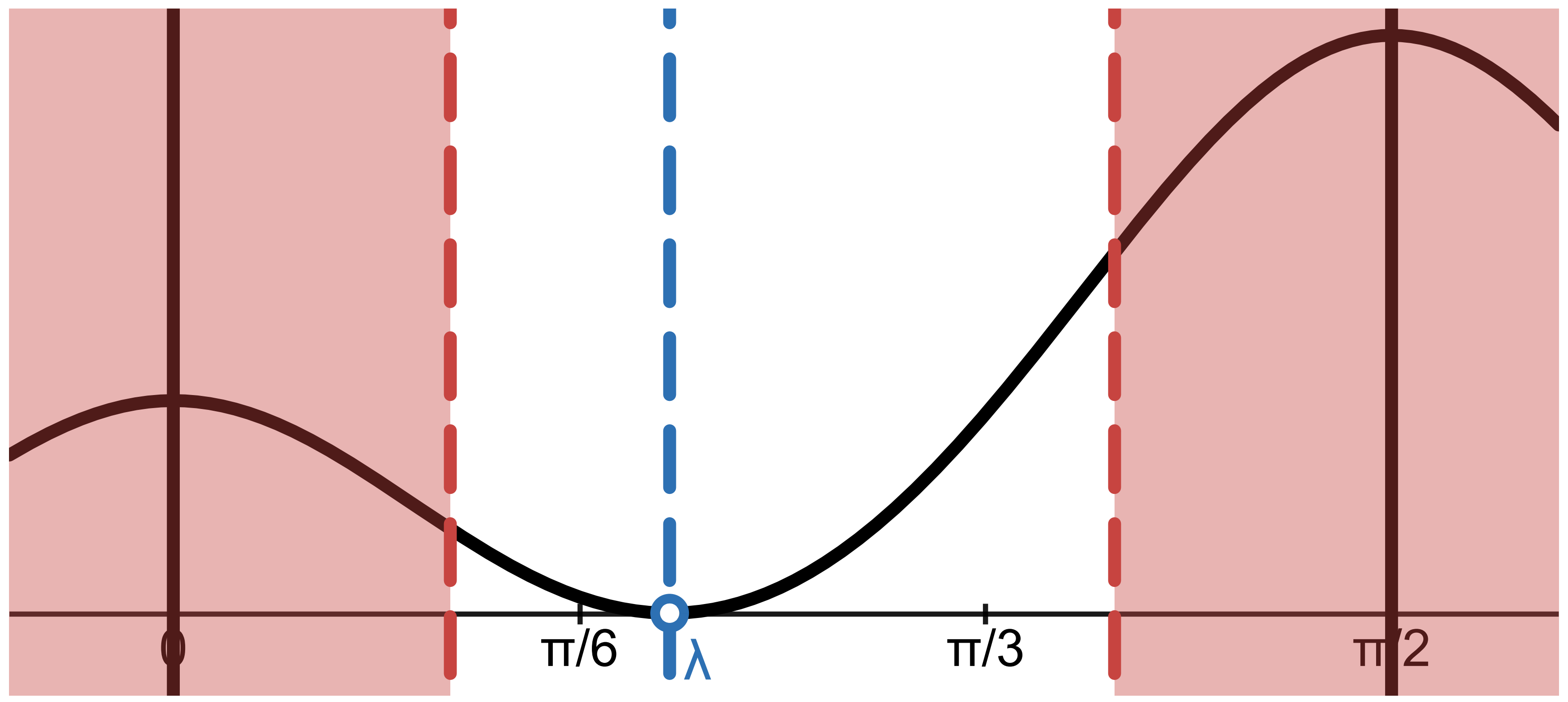


$$\tilde{\mathcal{L}}(\theta) = \frac{1}{N}\sum_{m\sim\tilde{p}_T}\left(Z_m - \cos(2\theta m)\right)^2,$$

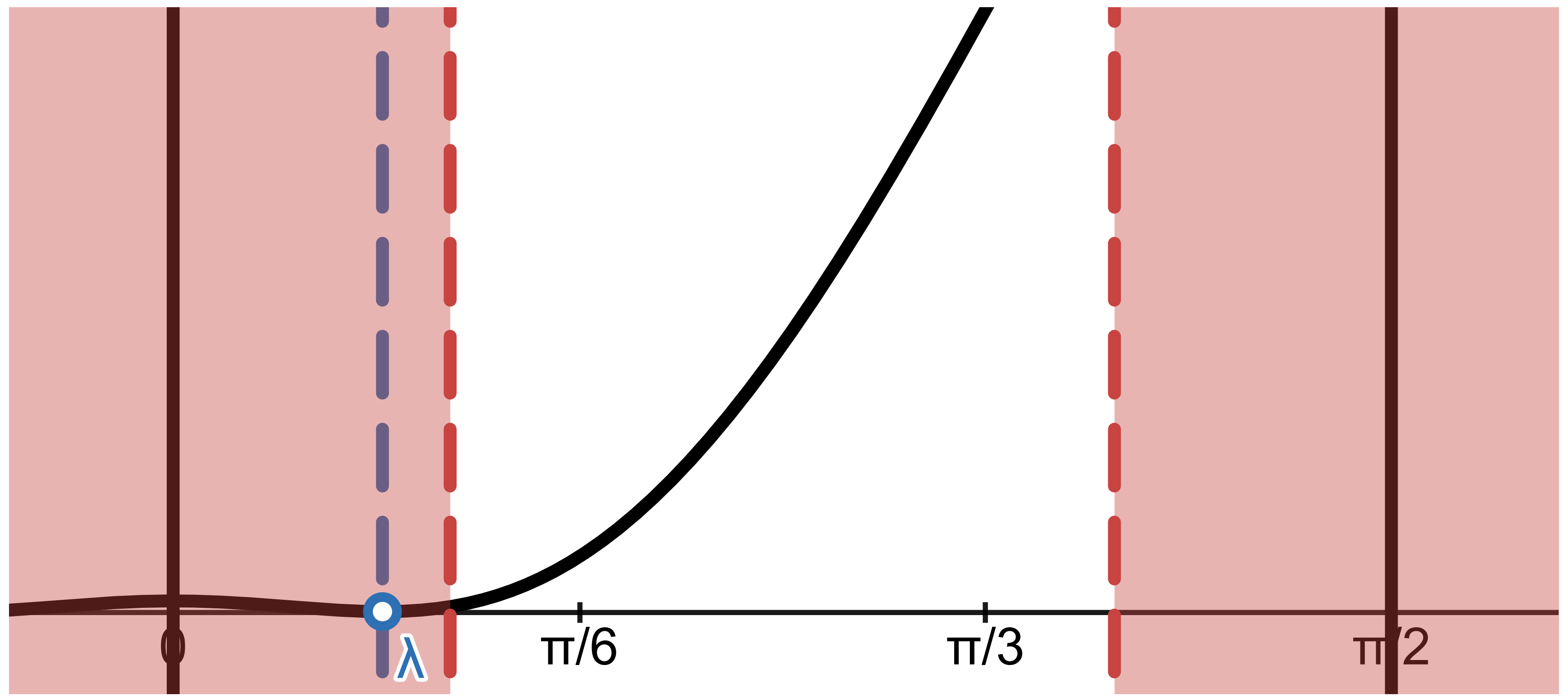
GLSAE fails at low depths — Only cosine signals



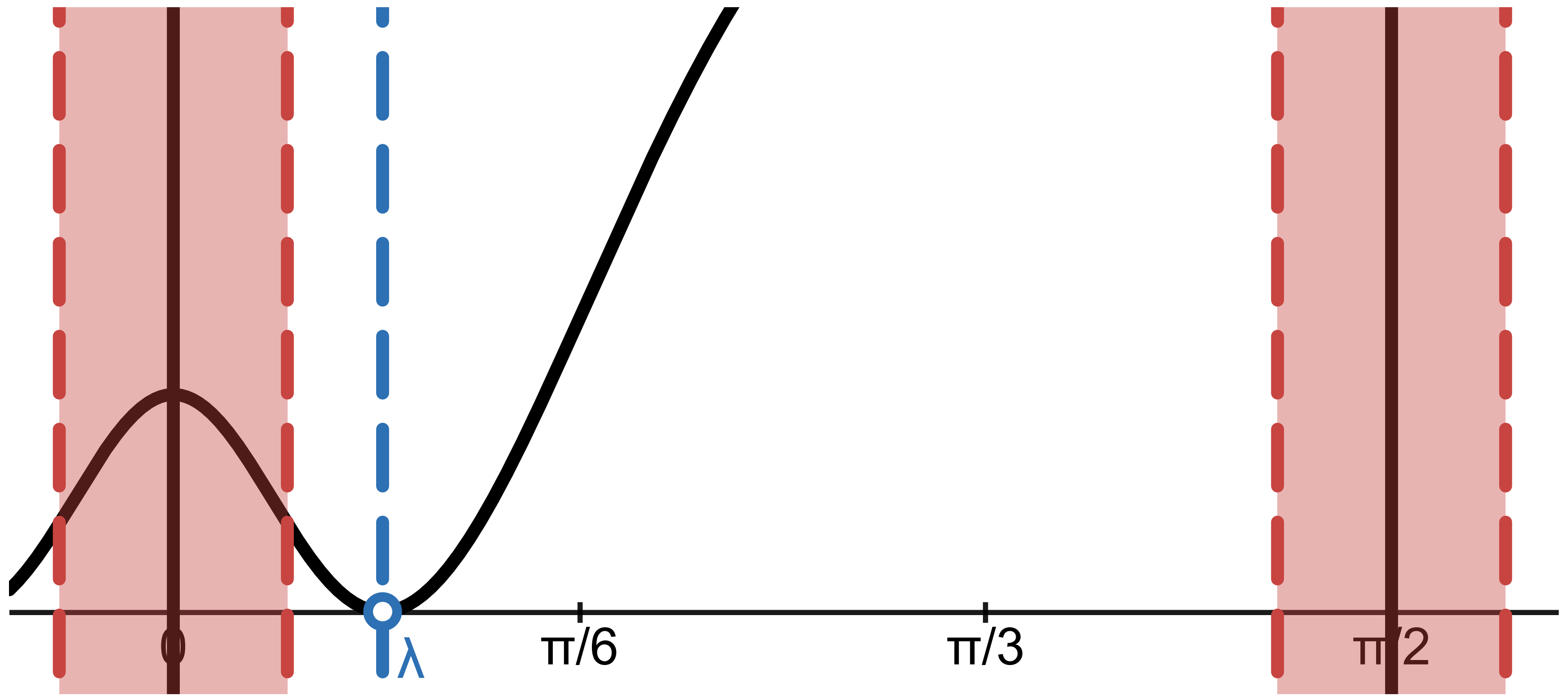
(a)



(b)

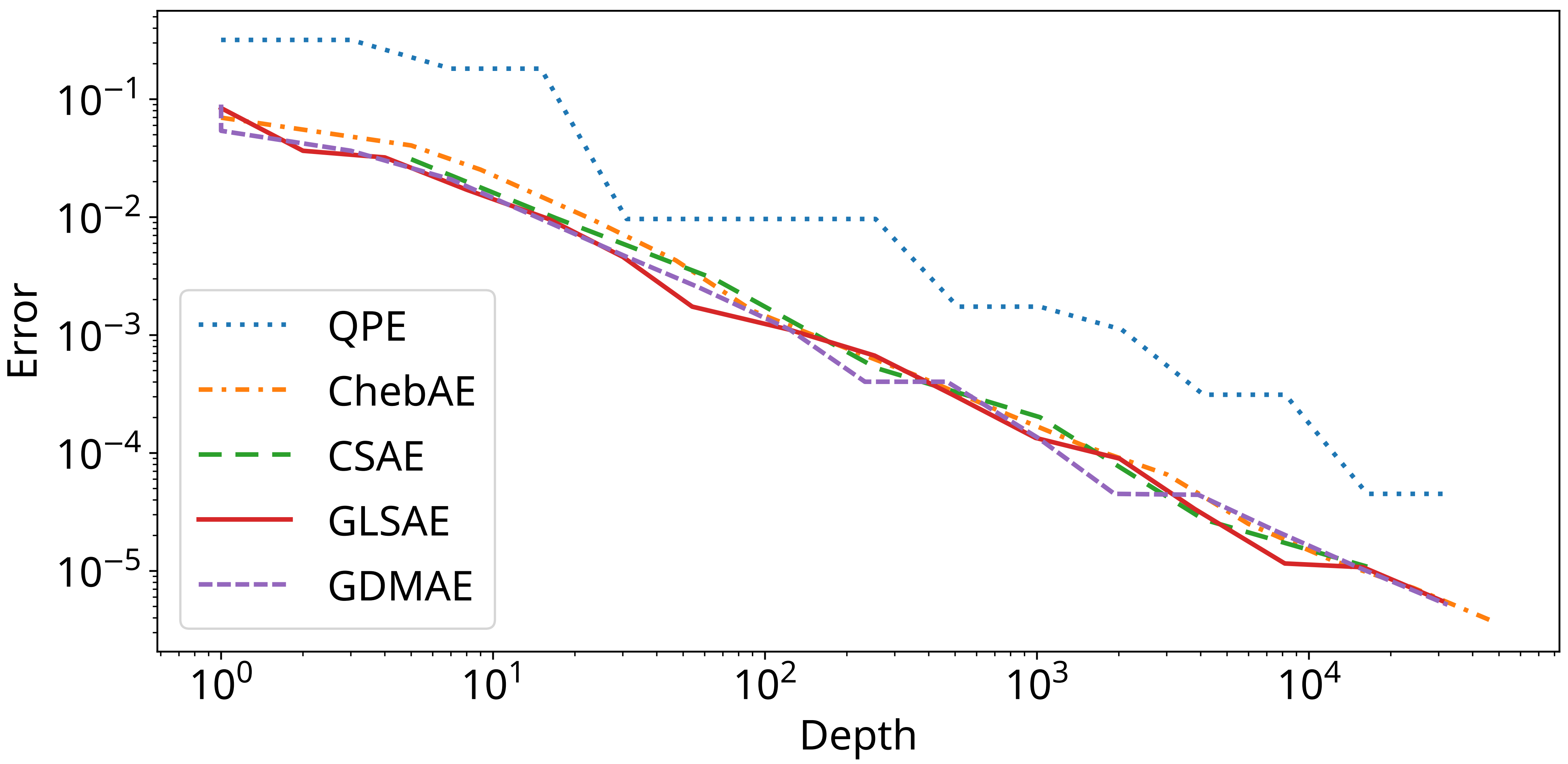


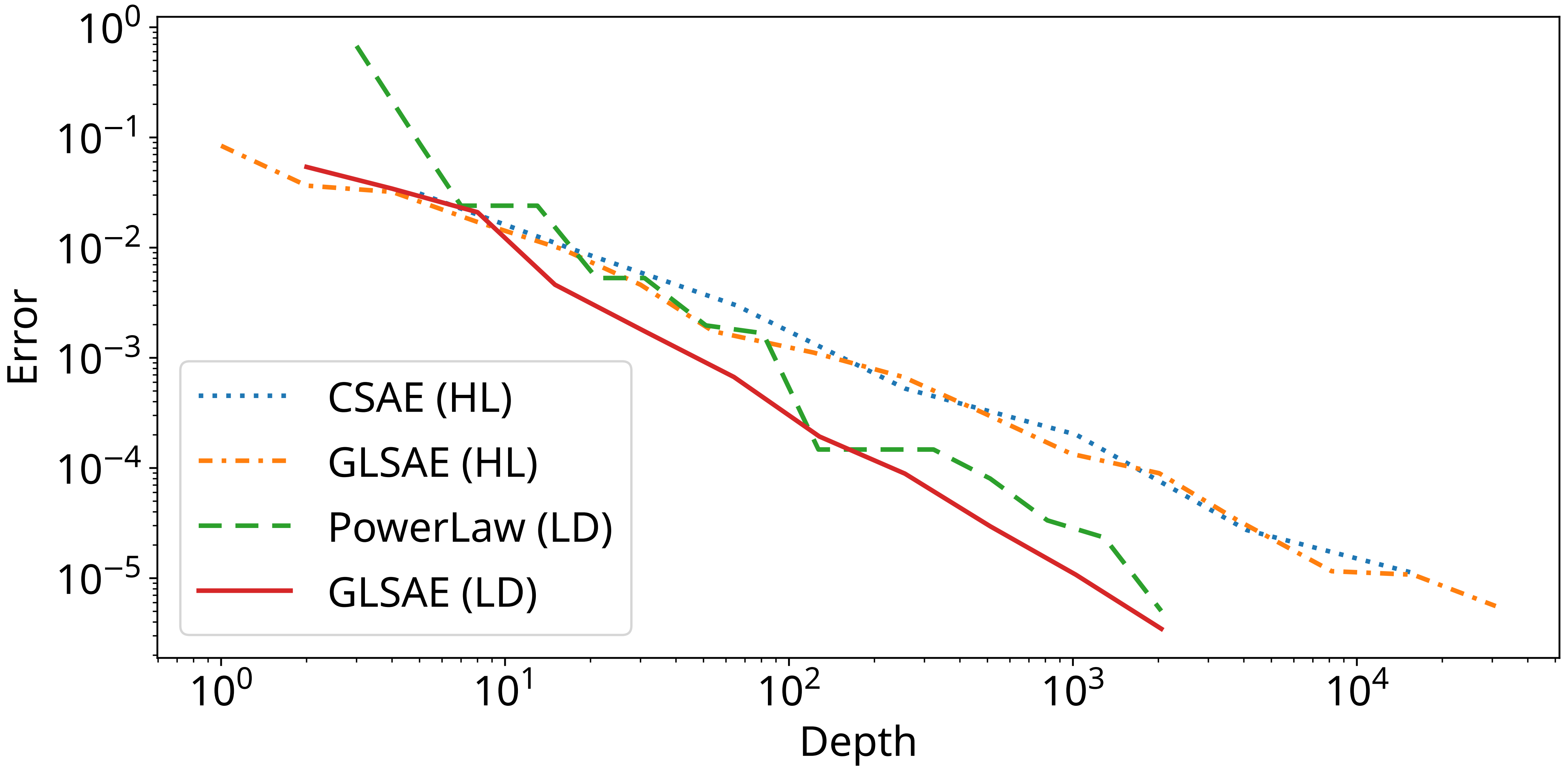
(c)

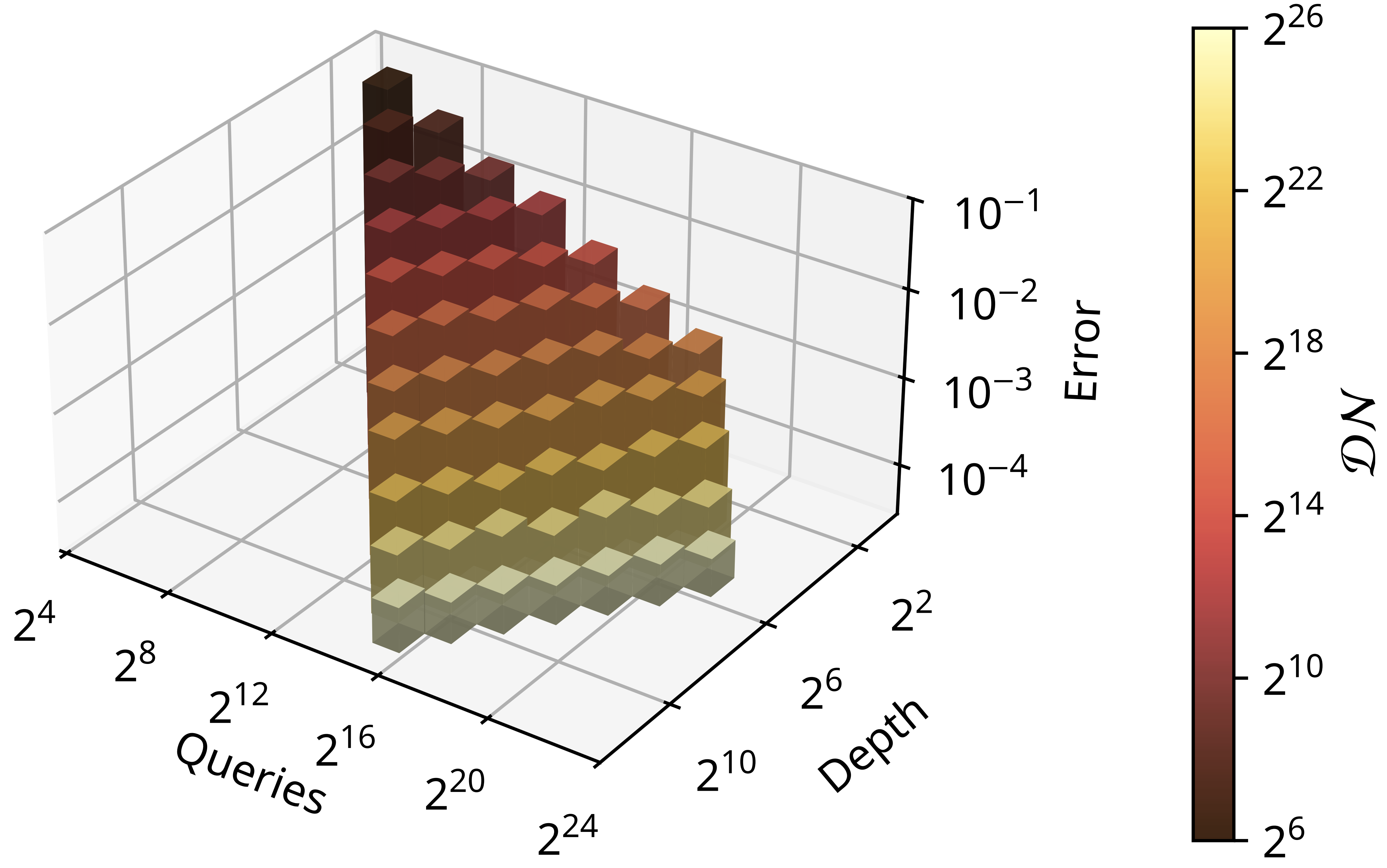


(d)

Numerical results

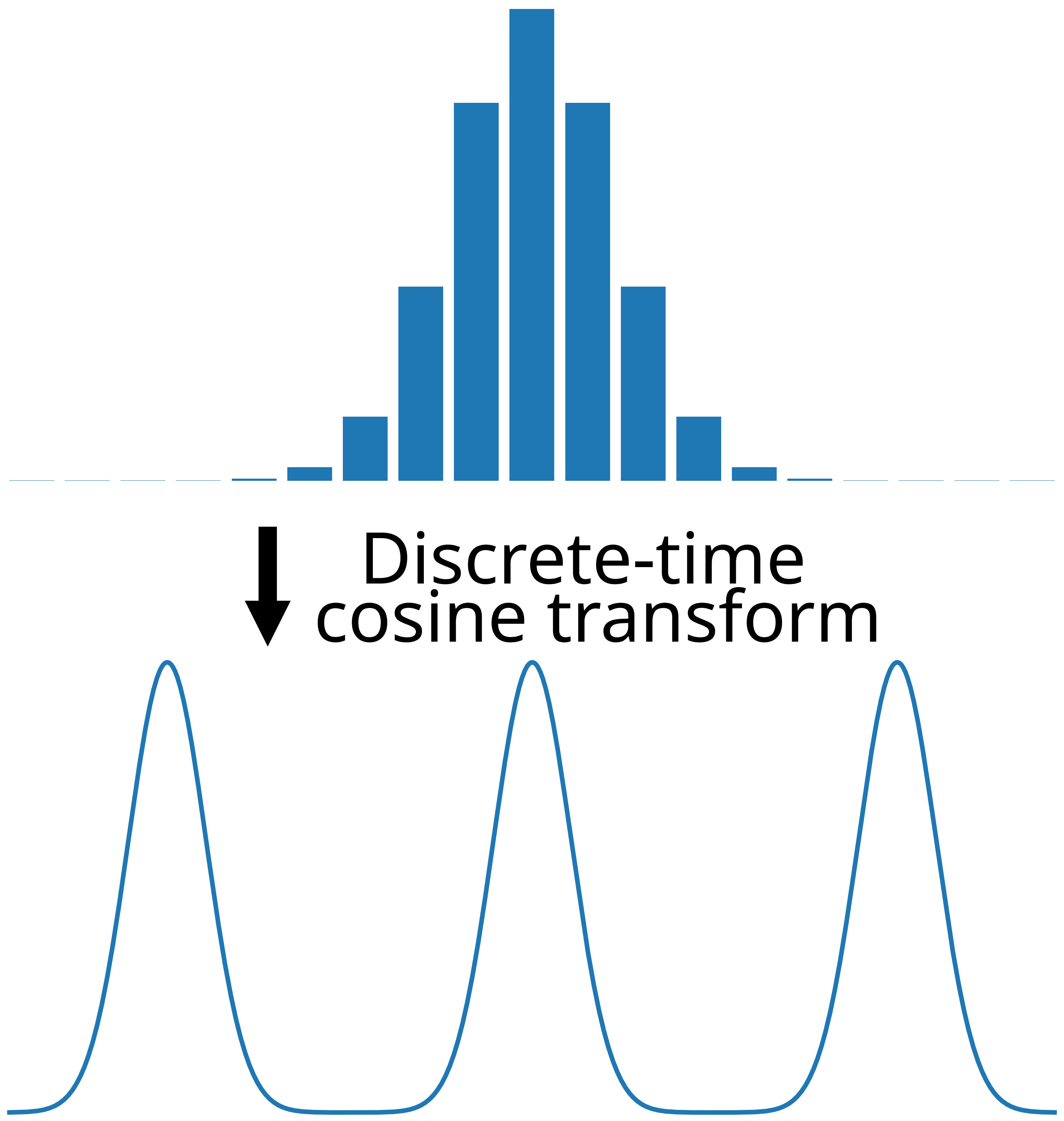






Proof ideas and strategy I

- Discrete Gaussian sample distribution + cosine signals
  - Discrete-time cosine transform of discrete Gaussian is a periodic Gaussian



Proof ideas and strategy II

- Lower quadratic bounds via strong convexity/concavity
- Upper quadratic bounds via smoothness
- Complete quadratic inequality bounds by including sampling error via Hoeffding bound and truncation errors to recover query-depth invariance